

# Production chains and general equilibrium aggregate dynamics<sup>\*</sup>

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## Abstract

Recent empirical studies reveal that monetary shocks can cause persistent fluctuations in aggregate output. In this paper, we propose a mechanism to help generate such persistence. Our dynamic stochastic general equilibrium model features a vertical input-output structure, with staggered price contracts at each stage of production. Working through the input-output relations and the timing of firms' pricing decisions, the model generates persistent fluctuations in aggregate output and the observed patterns of price dynamics following a monetary shock. Output responses are more persistent, the greater the number of stages of production, and the larger the share of intermediate inputs. With a sufficient number of stages, the persistence is arbitrarily large if the share of intermediate inputs is one at all but finitely many stages.

*Key words:* Production chains; Staggered price contracts; Persistence; Monetary policy

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## 1. Introduction

An order for a new computer often initiates a chain of orders for parts. When the order arrives at a computer vendor's desk, the vendor will start contacting suppliers of microchips, processors, hard-drives, monitors, and operating systems. The monitor maker will then contact suppliers of plastic, glass, and electronic components; and the plastic maker will respond by sending out orders to its own suppliers, and so on. The computer itself, once made, is frequently used as an intermediate input in the production of other goods.

The production of a final good typically goes through multiple stages of processing. A thesis of this paper is that the multi-stage structure of production can be important for explaining the relationship between money and aggregate economic activity. We show that the vertical input-output structure helps generate persistent fluctuations in aggregate output and the observed patterns of price dynamics following a monetary shock.

It is an old idea that, in an industrialized economy the relationship between money, prices, and output is tied to the interdependence of firms at different stages of production. The idea has been presented at least since Means (1935). Here we quote Basu (1995):

[Means] presented evidence that different industries had very different patterns of price changes versus quantity changes in the Great Depression. Means showed that simple goods, such as agricultural products, declined heavily in price, while their quantity was almost unchanged. Complex manufactured goods, on the other hand, showed the opposite pattern, with small price changes and consequently huge declines in the quantity of sales. Crude manufactured goods fell somewhere in between.

The evidence presented by Means (1935) has led many to conjecture that there are connections between an input-output structure and aggregate fluctuations. For example, Gordon (1990) considers "the input-output table as an essential component in the description of price stickiness." There is a growing literature of multi-sector models which are intended to explain the transmission of business cycle shocks through a horizontal round-about input-output structure within a single stage of production. This literature includes Long and Plosser (1983, 1987), Hornstein and Praschnik (1997), Horvath (1998, 2000) and Duper (1999), who focus on real shocks; and Basu (1995) and Bergin and Feenstra (2000), who focus on monetary shocks. On the other hand, recent studies confirm Means's observation on the patterns of price changes at different stages of production. For example, Clark (1999) studies a broad range of data sets and finds that "prices at early stages of production respond more to a monetary policy shock than do prices at

subsequent stages of production.” (See, also, Gordon (1981), Blanchard (1987), and Hanes (1999).) Yet, little theoretical work has been done to investigate the shock transmission mechanism embodied in a vertical input-output structure, with the notable exception of Blanchard (1983).

Blanchard (1983) shows that a simple reduced-form model incorporating a vertical production chain with prices staggered across different stages of processing can generate patterns of price changes similar to those noted by Means (1935). He was concerned with explaining the sluggish adjustment of the price level. More recently, another empirical fact has attracted much attention: the persistent response of aggregate output to a monetary shock (e.g., Gali (1992) and Christiano, Eichenbaum, and Evans (1999)). Chari, Kehoe, and McGrattan (2000) demonstrate the challenge facing traditional models of staggered price contracts in the spirit of Taylor (1980) in accounting for the observed output persistence in a general equilibrium framework. Motivated by this challenge, various mechanisms have been proposed, most of which focus on introducing factor market frictions in the baseline model of Chari, et al. (2000)<sup>1</sup>

In this paper, we propose a new mechanism that helps meet the challenge posted by Chari, et al. (2000) while explaining the observed patterns of price dynamics. We construct a model in which the production of a final good goes through multiple stages of processing, as in Blanchard (1983), but in which individuals optimize. In our model, a firm at the first stage of production uses labor as an input, while a firm at a later stage uses both labor and goods produced at the previous stage. Firms behave as imperfect competitors in their output markets and are price-takers in their input markets. Labor market is perfectly competitive. A representative household consumes a basket of goods produced at the final stage and supplies labor to firms at all stages. To obtain analytical solutions in a system of log-linearized equilibrium conditions, we abstract from capital accumulation. This simplification does not alter our basic conclusions.<sup>2</sup> To generate real effects of a monetary shock, we assume that pricing decisions are staggered among firms within each stage (e.g., Taylor (1980, 1999)) and we derive firms’ optimal pricing decision rules in a standard monopolistic competition framework (e.g., Blanchard and Kiyotaki (1987)). Working through the input-output relations among industries across stages and the timing of pricing decisions among firms within each stage, the model generates persistent responses of aggregate output following a monetary shock and replicates the patterns of price adjustments similar to those documented by Clark (1999) and others. The responses of aggregate output are more persistent, the greater the number of

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<sup>1</sup>See, for example, Huang and Liu (1998) and Gust (1998). Other recent general equilibrium models with staggered nominal contracts include Bergin and Feenstra (2000), Cho, Cooley, and Phaneuf (1997), Christiano, Eichenbaum, and Evans (1997), Erceg (1997), Erceg, Henderson, and Levin (2000), Kiley (1997), Kim (2000), Koenig (1997), Rotemberg (1996), Rotemberg and Woodford (1997), and Yun (1996).

<sup>2</sup>See Huang and Liu (1999) for a model of production chains with capital accumulation.

stages of production, and the larger the share of intermediate inputs. With a sufficient number of stages, the persistence is arbitrarily large if the share of intermediate inputs is one at all but finitely many stages.

The vertical input-output structure is an essential feature of our model in explaining the dynamics of prices and aggregate output following a monetary shock. In a model with a single stage of production (and thus without the vertical input-output structure), prices adjust quickly and there is no real effect of money beyond the initial contract duration (e.g., Chari, et al. (2000)). This is so because the shock leads to a quick change in the wage rate and hence in the marginal cost for all firms. In our model with multiple stages of production, firms at more advanced stages of processing face smaller changes in their marginal cost and thus have smaller incentives to change their prices than do firms at less advanced stages. Consequently, movements in prices are dampened through the production chain and the response of aggregate output dies out gradually.

The intuition behind the price dampening mechanism of the production chain is as follows. Following a monetary shock, the marginal cost for firms at the first stage immediately changes, forcing them to change their prices fully whenever they have the chance to renew contracts. But firms at the second stage do not face a full change in their marginal cost, because the marginal cost of these firms is partly determined by the price index of the first-stage goods and the price index records both the prices newly adjusted and the prices fixed by previous contracts. Thus, these firms do not have an incentive to adjust their prices fully even if they have the chance to renew contracts. Likewise, firms at the third stage face an even smaller change in their marginal cost and thus have an even smaller incentive to adjust their prices, and so on. Therefore, the multi-stage input-output structure creates a “real rigidity” in the sense of Ball and Romer (1990). When the number of stages of production gets larger, the adjustment of the price level becomes more sluggish and the response of aggregate output becomes more persistent.

The input-output structure in our model corresponds in a broad sense to the input-output relations identified by Clark (1999), and our model is able to replicate his findings about the basic behaviors of prices across different stages of production following a monetary shock. For instance, our model can explain why the prices of crude materials are more sensitive to a monetary shock than are the prices of primary goods, which in turn are more sensitive than are the prices of semi-finished goods and finished goods, and so on, a pattern documented by Clark (1999). Our model also predicts that, depending on the length of the production chain, movements in the price level such as the CPI can lag behind movements in prices at early stages of production. It thus provides justification for the practice of policy makers and forecasters looking for signs of impending rise in the general price level by concentrating on price movements in some price-sensitive sectors such as the crude material sector.

Our result that the magnitude of price stickiness is increasing in the number of stages of production is similar to that of Blanchard (1983), but for different reasons. In his model, pricing decisions are staggered across different stages and firms within each stage are homogeneous. Basu (1995) points out that, “if the pricing decision in Blanchard’s model were made state-dependent then, since the ‘first good’ is made without intermediate goods, there would be no increase in price rigidity regardless of the number of stages of production.” Basu’s (1995) criticism does not apply to our model since here pricing decisions are staggered among firms *within* each stage. Under a state-dependent pricing rule, firms at each stage do not have incentives to synchronize their pricing decisions provided that they face different price adjustment costs (e.g., Dotsey, et al. (1997, 1999)). As long as firms at *some* stages do not synchronize, the effects of a monetary shock on price adjustments will be dampened through the production chain.

The assumption that pricing decisions are staggered is supported by empirical evidence (e.g., Taylor (1999)). Yet, answering the question why there is staggering rather than complete synchronization is beyond the scope of this paper. In the literature, some progress has been made on this issue. Dotsey, et al. (1997) show that introducing heterogeneity of menu costs across firms can result in endogenous staggering. Ball and Romer (1989) demonstrate that staggering is an equilibrium outcome if there are firm-specific shocks that arrive at different time for different firms. Ball and Cecchetti (1988) show that, with imperfect information, firms cannot distinguish between aggregate demand shocks and firm-specific shocks, and thus do not have an incentive to synchronize. Gordon (1990) argues that, in a world with imperfect information, the complexity of the input-output table makes it unlikely for firms to synchronize, since “the typical firm has no idea of the identity of its full set of suppliers when all the indirect links within the input-output table are considered. . . . [T]he sensible firm just waits by the mailbox for news of cost increases and then . . . passes them on as price increases.” Clearly, incorporating these elements to make staggering endogenous will make a model intuitively more appealing. But the basic mechanism by which the production chain transmits monetary shocks will stay the same.

We describe the model in Section 2, present the results in Section 3, and conclude the paper in Section 4. The proofs of the analytical results are sketched in the Appendix.

## **2. The model**

In the model economy, the production of a final consumption good requires  $N$  stages of processing, from crude material to intermediate goods, then to more advanced goods, and so on. At each stage, there is a continuum of firms indexed in the interval  $[0, 1]$  producing differentiated goods. The production at stage 1 requires homogeneous labor services provided by a representative household, and the production

at stage  $n \in \{2, \dots, N\}$  uses both labor and goods produced at stage  $n - 1$ . Figure 1 illustrates this input-output structure.

In each period  $t$ , there realizes a shock  $s_t$ . The history of events up to date  $t$  is  $s^t \equiv (s_0, \dots, s_t)$ , with probability  $\pi(s^t)$ . The initial realization  $s_0$  is given.

The representative household is infinitely lived and has the following utility function

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi(s^t) \left[ \ln C(s^t) + \Phi \ln \left( \frac{M(s^t)}{\bar{P}_N(s^t)} \right) - \Psi L(s^t) \right], \quad (1)$$

where  $\beta \in (0, 1)$  is a subjective discount factor,  $C(s^t)$  is consumption,  $M(s^t)$  is nominal money balances,  $L(s^t)$  is labor hours, and  $\bar{P}_N(s^t)$  is a price index for goods produced at the final stage and thus corresponds to the price level of the economy. The consumption good is a Dixit-Stiglitz (1977) composite of the final-stage goods. Specifically, we have

$$C(s^t) = \left[ \int_0^1 Y_N(i, s^t)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \equiv Y_N(s^t), \quad (2)$$

where  $Y_N(i, s^t)$  is a type  $i$  good produced at stage  $N$  and  $\theta$  is the elasticity of substitution between such goods. The household is endowed with one unit of time in each period and faces a sequence of budget constraints

$$\begin{aligned} \int_0^1 P_N(i, s^t) Y_N(i, s^t) di + \sum_{s^{t+1}} D(s^{t+1}|s^t) B(s^{t+1}) + M(s^t) \\ \leq W(s^t) L(s^t) + \Pi(s^t) + B(s^t) + M(s^{t-1}) + T(s^t), \end{aligned} \quad (3)$$

where  $P_N(i, s^t)$  is the price of a type  $i$  good produced at the final stage,  $B(s^{t+1})$  is a one-period nominal bond that costs  $D(s^{t+1}|s^t)$  dollars at  $s^t$  and pays off one dollar in the next period if  $s^{t+1}$  is realized,  $W(s^t)$  is a nominal wage,  $\Pi(s^t)$  is the household's claim to all firms' profits, and  $T(s^t)$  is a nominal lump-sum transfer from the government. The household maximizes its utility (1) subject to (2), (3), and a borrowing constraint  $B(s^t) \geq -\bar{B}$ , for some large positive  $\bar{B}$ , taking the wage rate and prices as given. The initial conditions  $M(s^{-1})$  and  $B(s^0)$  are given.

The price level  $\bar{P}_N(s^t)$  is given by  $\bar{P}_N(s^t) = \left[ \int_0^1 P_N(i, s^t)^{1-\theta} di \right]^{\frac{1}{1-\theta}}$ . Utility maximization implies that the expenditure on the composite consumption good is equal to the total expenditure on all types of goods produced at the final stage, that is,  $\bar{P}_N(s^t) Y_N(s^t) = \int_0^1 P_N(i, s^t) Y_N(i, s^t) di$ . The demand function for a type  $i$  good produced at stage  $N$  is

$$Y_N^d(i, s^t) = \left[ \frac{P_N(i, s^t)}{\bar{P}_N(s^t)} \right]^{-\theta} Y_N(s^t). \quad (4)$$

Thus, the more expensive is good  $i$  relative to other goods produced at stage  $N$ , the lower is the relative demand for  $i$ .

The production of each good at stage 1 uses labor services only, with a constant-returns-to-scale technology given by  $Y_1(i, s^t) = L_1(i, s^t)$ , where  $Y_1(i, s^t)$  is the output and  $L_1(i, s^t)$  is the labor input. The production of each good at stage  $n \in \{2, \dots, N\}$  requires labor and all goods produced at the previous stage as inputs according to the following technology

$$Y_n(i, s^t) = \left[ \int_0^1 Y_{n-1}(i, j, s^t)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta\gamma}{\theta-1}} L_n(i, s^t)^{1-\gamma}, \quad (5)$$

where  $Y_n(i, s^t)$  is the output of a stage- $n$  firm of type  $i$ ,  $Y_{n-1}(i, j, s^t)$  is the input supplied to  $i$  by a stage- $(n-1)$  firm of type  $j$ ,  $L_n(i, s^t)$  is the labor input used by  $i$ , and  $\gamma \in (0, 1)$  is the share of the composite of stage- $(n-1)$  goods in  $i$ 's production.

Firms behave as imperfect competitors in their output markets and are price-takers in their input markets. To generate real effects of monetary shocks, we assume that the pricing decisions of firms at each stage of production are staggered. We derive the optimal pricing decision rules within a monopolistic competition framework. To focus on the role of the production chain in propagating monetary shocks, we assume simple two-period staggered price contracts when deriving our analytical results. Under such contracts, in each period, half of the firms at each stage can set new prices for their outputs. Once a price is set, it remains effective for two periods, which is referred to as a ‘‘contract duration’’. We sort the indices of firms at each stage so that those indexed  $i \in [0, 1/2]$  set new prices in periods 0, 2, 4,  $\dots$ , and those indexed  $i \in (1/2, 1]$  set new prices in periods 1, 3, 5,  $\dots$ , and so on.

Upon the realization of  $s^t$ , if firm  $i$  at stage  $n \in \{1, \dots, N\}$  can set a new price, it chooses  $P_n(i, s^t)$  to solve

$$\text{Max} \quad \sum_{\tau=t}^{t+1} \sum_{s^\tau} D(s^\tau | s^t) [P_n(i, s^t) - V_n(i, s^\tau)] Y_n^d(i, s^\tau), \quad (6)$$

taking the unit cost  $V_n(i, s^\tau)$  and the demand schedule  $Y_n^d(i, s^\tau)$  as given.

The unit cost for a firm at stage 1 is simply the nominal wage rate, that is,

$$V_1(s^t) \equiv V_1(i, s^t) = W(s^t), \quad (7)$$

since labor is the only input at that stage. The unit cost for a firm at stage  $n \in \{2, \dots, N\}$  is derived from minimizing  $\int_0^1 P_{n-1}(j) Y_{n-1}(i, j) dj + W L_n(i)$  subject to (5), and is given by

$$V_n(s^t) \equiv V_n(i, s^t) = \tilde{\gamma} \bar{P}_{n-1}(s^t)^\gamma W(s^t)^{1-\gamma}, \quad (8)$$

where  $\tilde{\gamma} = \gamma^{-\gamma}(1 - \gamma)^{-(1-\gamma)}$ , and  $\bar{P}_{n-1}(s^t) \equiv \left[ \int_0^1 P_{n-1}(j, s^t)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$  is a price index for goods produced at stage  $n-1$ . Given the constant-returns-to-scale technologies, the unit cost is also the marginal cost and is firm-independent.

The demand schedule for a good produced at stage  $N$  is simply given by (4). The demand schedule for a good produced at stage  $n \in \{1, \dots, N-1\}$  is derived from the cost-minimization problems of firms at stage  $n+1$  and is given by

$$Y_n^d(i, s^t) = \left[ \frac{\gamma}{1-\gamma} \right]^{1-\gamma} \left[ \frac{P_n(i, s^t)}{\bar{P}_n(s^t)} \right]^{-\theta} \left[ \frac{\bar{P}_n(s^t)}{W(s^t)} \right]^{-(1-\gamma)} \tilde{Y}_{n+1}(s^t), \quad (9)$$

where  $\tilde{Y}_{n+1}(s^t) \equiv \int_0^1 Y_{n+1}(j, s^t) dj$  is a linear aggregate of goods produced at stage  $n+1$ .

Solving the profit-maximization problem (6) yields the optimal pricing decision rule

$$P_n(i, s^t) = \frac{\theta}{\theta-1} \frac{\sum_{\tau=t}^{t+1} \sum_{s^\tau} D(s^\tau | s^t) Y_n^d(i, s^\tau) V_n(s^\tau)}{\sum_{\tau=t}^{t+1} \sum_{s^\tau} D(s^\tau | s^t) Y_n^d(i, s^\tau)}, \quad (10)$$

where  $n \in \{1, \dots, N\}$ . Thus, the optimal price is a constant mark-up over a weighted average of a firm's marginal costs within its contract duration. The weights are normalized quantities of demand for the firm's output which, in light of (4) and (9), depend only on industry- and economy-wide variables. If the expected marginal costs rise, the firm will respond by raising its price.

A monetary authority injects newly created money into the economy via a lump-sum transfer to the household, that is,

$$T(s^t) = M^s(s^t) - M^s(s^{t-1}). \quad (11)$$

The money supply  $M^s(s^t)$  grows at a rate  $\mu(s^t)$  so that  $M^s(s^t) = \mu(s^t)M^s(s^{t-1})$ . We assume that  $\ln \mu(s^t)$  follows a stationary stochastic process.

An *equilibrium* for this economy consists of allocations  $\{Y_N(i, s^t)\}_{i \in [0,1]}$ ,  $L(s^t)$ ,  $M(s^t)$ , and  $B(s^{t+1})$  for the household, allocations  $\{L_1(i, s^t)\}_{i \in [0,1]}$  and prices  $\{P_1(i, s^t)\}_{i \in [0,1]}$  for firms at stage 1, allocations  $\{L_n(i, s^t)\}_{i \in [0,1]}$  and  $\{Y_{n-1}(i, j, s^t)\}_{i, j \in [0,1]}$ , and prices  $\{P_n(i, s^t)\}_{i \in [0,1]}$  for firms at stage  $n \in \{2, \dots, N\}$ , together with wage rate  $W(s^t)$ , bond prices  $D(s^{t+1} | s^t)$ , and price indices  $\{\bar{P}_n(s^t)\}_{n \in \{1, \dots, N\}}$ , that satisfy the following conditions: (i) taking the wage and prices as given, the household's allocations solve its utility maximization problem; (ii) taking the wage and all prices but its own as given, each firm's allocations and price solve its profit maximization problem; (iii) markets for labor, money, and bonds clear; (iv) money supply and transfer are as specified.

It is important to point that  $Y_N(s^t)$ , the composite of all goods produced at stage  $N$ , can be interpreted as aggregate output, corresponding to real GDP in the data (note that in our closed-economy model with

no capital or government spending, real GDP corresponds to aggregate consumption). To justify this interpretation, we first observe that the budget constraint (3) is binding in equilibrium for the utility function is strictly monotone. By imposing the money market clearing condition and the transfer process (11), we can cancel out the terms involving money balances and transfer in the budget equation. From the bond market clearing condition  $B(s^t) = 0$ , the terms involving nominal bonds drop out as well. With the equilibrium relation  $\bar{P}_N(s^t)Y_N(s^t) = \int_0^1 P_N(i, s^t)Y_N(i, s^t)di$ , the budget equation then reduces to

$$\bar{P}_N(s^t)Y_N(s^t) = W(s^t)L(s^t) + \Pi(s^t). \quad (12)$$

The left-hand side of (12) is the aggregate expenditure while the right-hand side is the total income, including total wage income and total equity income. The total wage income is the sum of the wage earnings across the  $N$  stages of production and the total equity income is the sum of the profits of firms across all stages. Thus, the right-hand side is also the aggregate value-added. It is clear from this equation that  $Y_N(s^t)$  corresponds to real aggregate output, or real GDP.

In what follows, we focus on a symmetric equilibrium in which firms in the same cohort make identical pricing decisions. In a symmetric equilibrium, firms are identified by the stage at which they produce and the time at which they can change prices. Thus, from now on we drop the indices  $i$  and  $j$  for individual firms, and let  $P_n(t)$  denote prices set at time  $t$  for goods produced at stage  $n \in \{1, \dots, N\}$ .

### 3. The results

In this section, we demonstrate that the model with a vertical input-output structure presented in Section 2 can generate the observed patterns of price movements at different stages of production and more importantly, persistent responses of aggregate output following a monetary shock. We show that the output persistence increases with the length of the production chain.

To gain insights into our findings, we derive analytical solutions to a log-linearized system of equilibrium conditions. We begin by reducing the equilibrium conditions to  $2N + 2$  equations, including  $N$  pricing equations, a labor supply equation, a money demand equation, and  $N$  equations defining price indices and the price level. We then log-linearize these equations around a deterministic steady state and use lowercase letters to denote the log-deviations of the corresponding level variables from their steady state values.<sup>3</sup>

The linearized pricing rule for firms at stage  $n \in \{1, \dots, N\}$  is

$$p_n(t) = \frac{1}{1 + \beta} [\gamma \bar{p}_{n-1}(t) + (1 - \gamma)w(t)] + \frac{\beta}{1 + \beta} \mathbf{E}_t [\gamma \bar{p}_{n-1}(t + 1) + (1 - \gamma)w(t + 1)], \quad (13)$$

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<sup>3</sup>We derive the equilibrium conditions and report the log-linearization process in a Technical Appendix, which is available from the authors upon request.

where  $\bar{p}_0(t)$  denotes  $w(t)$  and  $E_t$  is a conditional expectation operator. According to (13), a firm's optimal price is a weighted average of its expected marginal costs within the contract duration. The marginal cost for a firm at stage 1 is equal to the wage rate since labor is the only input at that stage. The marginal cost for a firm at stage  $n \in \{2, \dots, N\}$  is an average of the wage rate and the price index of goods produced at stage  $n - 1$  since the firm uses both labor and stage- $(n - 1)$  goods as inputs. If the marginal cost is expected to rise, the firm will respond by setting a higher price whenever it can renew its contract.

The following equation describes the labor supply decision of the household

$$w(t) = \bar{p}_N(t) + y_N(t). \quad (14)$$

According to (14), real wage is proportional to aggregate output.

The money demand equation is given by

$$\bar{p}_N(t) + y_N(t) = (1 - \beta)m(t) + \beta E_t[\bar{p}_N(t + 1) + y_N(t + 1)]. \quad (15)$$

Therefore, nominal GDP is a weighted average of money and expected future nominal GDP. The presence of the expectation terms in (15) reveals that the demand for money is interest-rate sensitive.

Finally, the price index for goods produced at stage  $n \in \{1, \dots, N\}$  is related to pricing decisions at that stage by the following relation

$$\bar{p}_n(t) = \frac{1}{2}p_n(t - 1) + \frac{1}{2}p_n(t). \quad (16)$$

Under staggered contracts, the price index records both the prices set in the current period and those set in the previous period. The lagged price enters (16) because each contract lasts for two periods. Note that the price level is equal to the price index for goods produced at stage  $N$ .

The equilibrium conditions are fully described by (13)-(16). To focus on the role of the input-output structure in generating the observed patterns of price dynamics and output persistence, we assume that there is no serial correlation in the money growth process. In particular, we assume that the supply of money follows a random walk process, i.e.,  $m(t) = m(t - 1) + \epsilon(t)$ , where  $\epsilon(t)$  is a white noise disturbance corresponding to the money growth rate. Suppose that there is a one percent shock to the money growth rate in period 0, that is,  $\epsilon(0) = 1$  and  $\epsilon(t) = 0$  for all  $t \geq 1$ . We compute the impulse response functions to determine how the shock is divided between movements in the price level and in aggregate output. For this purpose, we focus on a perfect foresight equilibrium and drop the expectation operator  $E_t$ . The following proposition partially characterizes the equilibrium.

**Proposition 3.1:** *There is a unique perfect foresight equilibrium in which*

$$w(t) = 1, \quad t \geq 0, \quad (17)$$

$$p_n(t) = 1, \quad t \geq n - 1, \quad n \in \{1, \dots, N\}, \quad (18)$$

$$\bar{p}_n(t) = 1, \quad t \geq n, \quad n \in \{1, \dots, N\}, \quad (19)$$

$$y_N(t) = 0, \quad t \geq N, \quad (20)$$

for all  $N \geq 1$ .

Proposition 3.1 shows that the wage rate immediately rises by a full scale after the shock, and individual prices and the price index of stage  $n$  increases fully when the  $n^{\text{th}}$  and the  $(n + 1)^{\text{th}}$  periods arrive, respectively, as illustrated by Figure 2 (note that, in our model, period 0 is the first period). The proposition also shows that the price level (i.e., the price index at stage  $N$ ) adjusts fully and aggregate output goes back to steady state in the  $(N + 1)^{\text{th}}$  period, as illustrated by Figure 4.

The result in Proposition 3.1 in particular implies that the real effect of money does not go beyond the initial contract duration if there is only a single stage of production (i.e., if  $N = 1$ ). Following the shock, the wage rate and hence the marginal cost for firms at the first stage rise immediately, forcing these firms to raise their prices fully whenever they can set new prices. Upon the expiration of the initial price contracts, all firms have had a chance to adjust prices and the price index of the first-stage goods, which is also the price level in an economy with a single stage of production, is entirely composed of fully raised prices, and thus rises fully as well. Clearly, to generate persistent responses of aggregate output, sluggish adjustments in the price level are necessary. We now demonstrate that, when there are multiple stages of production, price adjustments are indeed sluggish.

**Proposition 3.2:** *In the perfect foresight equilibrium, the strict inequalities*

$$p_{n+1}(t) < p_n(t), \quad 0 \leq t \leq n - 1, \quad (21)$$

$$\bar{p}_{n+1}(t) < \bar{p}_n(t), \quad 0 \leq t \leq n, \quad (22)$$

hold for all  $n \in \{1, \dots, N - 1\}$ , and for all  $N \geq 2$ .

According to Proposition 3.2, the effects of the shock on prices are extenuated through the production chain so that price changes at later stages are smaller and less rapid than are price changes at earlier stages. Figure 2 demonstrates this pattern of movements in pricing decisions and price indices across different stages of production, and Table 1 provides a numerical illustration. We plot in Figure 3 the impulse

response functions of pricing decisions and price indices at different stages of production for the case with 4 cohorts of price-setting firms at each stage. The adjustments in prices display a similar pattern.

The key to understanding the pattern of price adjustments across different stages of production is to see how the effects of the shock on marginal costs diminish from earlier to later stages. Following the shock, firms at stage 1 face a full rise in marginal cost and consequently raise their prices fully whenever they can renew contracts. Firms at stage 2 do not immediately face a full rise in their marginal cost because the marginal cost of these firms is an average of the wage rate and the price index of stage-1 goods. This price index is an average of the prices newly adjusted and the prices fixed by contracts and, therefore, does not rise fully in the impact period. Facing a partial increase in marginal cost, firms at stage 2 choose not to raise their prices fully even if they can set new prices. At the end of the initial contract duration, the price index of stage-1 goods rises fully, so does the marginal cost for firms at stage 2. Thus, those firms that can renew contracts do choose to adjust their prices accordingly. Yet, the price index of stage-2 goods does not rise fully because it is an average of the prices newly adjusted and the prices partially adjusted in the impact period. When there are more than two stages of production, firms at stage 3 face even smaller changes in their marginal cost, and thus have even smaller incentives to adjust their prices, and so on.

Given that  $\bar{p}_N(t)$  is the price level for the case with  $N$  stages of production, (22) implies that, as the length of the production chain grows, movements in the price level become smaller on a period-by-period basis and the price level does not rise fully for longer periods of time, as illustrated by our numerical results in Table 1. This finding, coupled with (14) and (17), indicates that movements in aggregate output are larger on a period-by-period basis and are longer-lasting, the greater the number of stages of production. We summarize this result in the following proposition.

**Proposition 3.3:** *In the perfect foresight equilibrium, the strict inequality*

$$y_{N+1}(t) > y_N(t), \quad 0 \leq t \leq N, \quad (23)$$

*holds for all  $N \geq 1$ .*

The monotone relation between the response of aggregate output and the number of stages of production is illustrated by Table 2. To get a better quantitative feel, we plot in Figure 4 the impulse response functions of aggregate output and of the price level for various  $N$ . The figure shows that, while the response of the price level is decreasing in  $N$  on a period-by-period basis and the price level rises only partially in the first  $N$  periods after the shock, the response of aggregate output is increasing in  $N$  on a period-by-period basis and aggregate output stays above its steady state value for  $N$  periods. To test the

robustness of this result, we simulate our model for different number of cohorts of price-setting firms and obtain similar monotone relations. Figure 5 displays the impulse response functions of the price level and aggregate output for various  $N$  in the case with 4 cohorts of price-setting firms, which further confirm the result.

The result in Proposition 3.3 opens the way for the production chain to generate persistent real effects of the shock. Yet, to have a larger magnitude of output persistence also requires higher auto-correlations in the response of the output so that the initial response dies out more gradually. Based on this idea, we measure the magnitude of output persistence by the ratio of its response in period  $t$  to that in period  $t - 1$ , for all  $t \geq 1$  such that the ratio is well-defined.<sup>4</sup> Output persistence is monotone in the number of stages of production if the ratio of the response of aggregate output in period  $t$  to that in period  $t - 1$  is increasing in  $N$ .

**Proposition 3.4:** *In the perfect foresight equilibrium, the strict inequality*

$$\frac{y_{N+1}(t)}{y_{N+1}(t-1)} > \frac{y_N(t)}{y_N(t-1)}, \quad 1 \leq t \leq N, \quad (24)$$

*holds for all  $N \geq 1$ .*

Proposition 3.4 establishes the monotone relationship between the persistence and the length of the production chain. In other words, the greater the number of stages of production, the more persistent is the response of aggregate output to the shock. We plot in Figure 6 our general persistence measure as a function of time for various  $N$ . Clearly, the magnitude of persistence is increasing in  $N$ .

Our general measure of persistence nests as special cases two persistence measures often used in the literature. One is the ratio of the output response at the end of the initial contract duration to that in the impact period (i.e., the “contract multiplier”). The other is the number of periods it takes for output to return to half of the level of its initial response (i.e., the “half-life”). In light of Proposition 3.4, both the contract multiplier and the half-life increase with  $N$ . For example, when the share of intermediate inputs is ninety percent as in Basu (1995) and Bergin and Feenstra (2000), the contract multiplier increases from 0 to 0.35 and then to 0.50, as the number of stages of production grows from one to four and then to twelve (see the last row of Table 2). Therefore, our model with twelve stages of production generates the same

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<sup>4</sup>To see why this measure corresponds to the first order auto-correlation, consider an AR(1) process  $x(t) = \rho x(t-1) + e(t)$ ,  $t \geq 0$ , with an initial condition  $x(-1) = 0$ . If  $e(0) = 1$  and  $e(t) = 0$  for all  $t \geq 1$ , then the ratio  $x(t)/x(t-1)$  for  $t \geq 1$  measures the magnitude of the serial correlation in  $x$  in response to the  $e$  shock.

magnitude of the contract multiplier as in Bergin and Feenstra (2000), which does so via a non-linear interaction between non-CES preferences and a round-about input-output structure.

To illustrate our finding that the initial response of aggregate output dies out more gradually as the length of the production chain grows longer, we plot in Figure 7 the normalized impulse response function of aggregate output with respect to its initial response for various  $N$ . For robustness, we display such normalized output response for both our baseline model with two cohorts of price-setting firms and the case with four cohorts of price-setting firms at each stage of production. In both cases, our basic conclusion is evident.

It is of natural interest to ask: How long a way can the production chain go in generating real persistence? We provide an answer to this question by deriving the impulse response function of aggregate output for the case with a sufficient number of stages of production.

**Proposition 3.5:** *In the perfect foresight equilibrium, the equality*

$$\lim_{N \rightarrow \infty} y_N(t) = \frac{(1 + \beta)\xi^t}{2(1 + \beta) - (1 + \beta\xi)\gamma} \quad (25)$$

holds for all  $t \geq 0$ , where

$$\xi = \frac{(1 + \beta)(2 - \gamma) - \sqrt{(1 + \beta)^2(2 - \gamma)^2 - 4\beta\gamma^2}}{2\beta\gamma}. \quad (26)$$

With sufficiently many stages, the ratio of output response in a given period to that in the previous period is equal to  $\xi$ , as implied by (25). That is,  $\xi$  corresponds to our persistence measure for large  $N$ . Equation (26) implies that  $\xi$  is an increasing function of  $\gamma$ , the share of intermediate inputs in production at each stage. If the share is one at all but finitely many stages, then  $\xi = 1$ . In this case, we obtain arbitrary real persistence in the sense that the price level does not change and aggregate output carries the full burden of adjustment.

#### 4. Conclusion

We have shown that a model with a vertical input-output structure and staggered price contracts at each stage of production can generate persistent fluctuations in aggregate output and the observed patterns of price dynamics following a monetary shock. Output responses are more persistent, the greater the number of production stages, and the larger the share of intermediate inputs. With a sufficient number of stages, the persistence is arbitrarily large if the share of intermediate inputs is one at all but finitely many stages.

Our results do not hinge upon the assumption that the production of a good at a given stage uses *all* goods produced at the previous stage (along with labor services). This assumption is made only for analytical convenience. To dampen the fluctuations of marginal costs across different stages, what matters is that the down-stream firms supplying inputs to an up-stream firm do not change their prices simultaneously. It does not matter whether the input-supplying firms constitute all or just part of the firms at the same stage.<sup>5</sup>

That said, it is important to emphasize that we have merely identified the vertical input-output structure as one contributing mechanism that helps generate the observed persistent responses of aggregate output following a monetary shock. We do not claim that it is the only one—it is clearly not. There are other important contributing mechanisms identified in the literature, such as a horizontal round-about input-output structure (e.g., Basu (1995) and Bergin and Feenstra (2000)) and labor market rigidity (e.g., Huang and Liu (1998)). In the present paper, we abstract from these mechanisms in order to isolate the role of the vertical production chain in generating persistence. In our model with competitive labor markets, labor costs change rapidly in response to the shock, creating an incentive for quick adjustments in prices. Incorporating labor market rigidity will dampen the fluctuations in labor costs and, therefore, will make changes in the price level more sluggish and movements in aggregate output more persistent. A sensible quantitative model that aims at matching its statistics to the data should at least take into account all these monetary transmission mechanisms, should incorporate capital accumulation subject to adjustment costs, and should calibrate the shares of labor, capital, and intermediate inputs at each stage of production.

Needless to say, in a model like this, we also need to calibrate the number of stages of production (the  $N$  in the present model), which calls for a detailed examination of the input-output table. In light of our finding that the input-output structure is a potentially powerful mechanism in propagating monetary shocks, an empirical investigation of the input-output table should be elevated to the top of the research agenda. Casual observations suggests that  $N$  is likely to be large. On this, Gordon (1990) pictures the world as “a gigantic  $n \times n$  matrix, where  $n$  is measured in the thousands, if not the millions. . . . The gigantic matrix represents the real world, full of heterogeneous firms enmeshed in a web of intricate supplier-demander relationships.” In this web, the intricately made computer is perhaps just a tiny node.

## Appendix

In this appendix we sketch the major steps of our proofs of Propositions 3.1-3.5. The details of the proofs are available upon request from the authors.

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<sup>5</sup>For a more detailed discussion, see Huang and Liu (1999).

**Proof of Proposition 3.1:** Using (14), (15) and  $m(t) = 1$ , we obtain

$$w(t) = 1 + \beta^{-t}[w(0) - 1] \quad \forall t \geq 0. \quad (27)$$

Substituting (27) into (13) for the case with  $n = 1$  yields

$$p_1(t) = 1 + 2\beta^{-t}(1 + \beta)^{-1}[w(0) - 1] \quad \forall t \geq 0. \quad (28)$$

Substituting (16) and (27) into (13) leads to

$$p_n(t) = \frac{\gamma}{2(1 + \beta)}p_{n-1}(t - 1) + \frac{\gamma}{2}p_{n-1}(t) + \frac{\beta\gamma}{2(1 + \beta)}p_{n-1}(t + 1) + (1 - \gamma)p_1(t) \quad (29)$$

for all  $t \geq 0$  and  $n \in \{2, \dots, N\}$ . Using (28) and (29), we can show that

$$p_n(t) = 1 + 2\beta^{-t}(1 + \beta)^{-1}[w(0) - 1] \quad \forall t \geq n - 1 \quad \forall n \in \{1, \dots, N\}. \quad (30)$$

It follows from (16) and (30) that

$$\bar{p}_n(t) = w(t) \quad \forall t \geq n \quad \forall n \in \{1, \dots, N\}. \quad (31)$$

If  $w(0) > 1$  or  $w(0) < 1$ , then by (27), as  $t$  goes to infinity,  $w(t)$  diverges to plus or minus infinity at the rate of  $1/\beta$ , so does  $\bar{p}_N(t)$ , as implied by (31). These possibilities can be ruled out as in Obstfeld and Rogoff (1983, 1986). The hyper-inflationary path with  $\bar{p}_N(t) \rightarrow \infty$  cannot be an equilibrium for the household would suffer an infinite utility loss as real balances approach zero along such a path. The hyper-deflationary path with  $\bar{p}_N(t) \rightarrow -\infty$  cannot be an equilibrium either for it would violate the appropriate transversality condition with respect to real balances. Therefore,  $w(0) = 1$ , and there is a unique equilibrium in which  $w(t) = 1$  for all  $t \geq 0$  according to (27). That is, equation (17) holds. Equations (18) and (19) then follow from (30) and (31), respectively. Finally, (14) and (31) imply equation (20).□

**Proof of Proposition 3.2:** Using equations (13) and (16)-(19) we can verify (21) for  $n = 1$ . This establishes (21) if  $N = 2$ . Assume  $N > 2$ . Suppose that (21) holds for  $n$  with  $1 \leq n \leq N - 2$  and fix arbitrary  $t$  with  $0 \leq t \leq n$ . The induction hypothesis and (18) imply that

$$p_{n+1}(t - 1) \leq p_n(t - 1), \quad p_{n+1}(t) \leq p_n(t), \quad p_{n+1}(t + 1) \leq p_n(t + 1),$$

with at least one strict inequality. These inequalities, (17) and (29) imply that  $p_{n+2}(t) < p_{n+1}(t)$ , which completes the proof of (21). To prove (22), fix arbitrary  $n$  with  $1 \leq n \leq N - 1$  and  $t$  with  $0 \leq t \leq n$ . Relations (18) and (21) imply that

$$p_{n+1}(t - 1) \leq p_n(t - 1), \quad p_{n+1}(t) \leq p_n(t),$$

with at least one strict inequality. These inequalities and (16) imply that  $\bar{p}_{h+1}(t) < \bar{p}_n(t)$ , which establishes (22).  $\square$

**Proof of Proposition 3.3:** This is a direct corollary of (14), (17), and Proposition 3.2.  $\square$

**Proof of Proposition 3.4:** Using equations (13)-(14) and (16)-(20) we can verify (24) for  $N = 1, 2, 3$ .

Suppose that (24) holds for  $N \geq 3$ . Equations (13)-(14) and (16)-(17) imply that

$$y_{N+1}(t) = \begin{cases} \frac{\gamma}{2(1+\beta)}y_N(t-1) + \frac{\gamma}{2}y_N(t) + \frac{\beta\gamma}{2(1+\beta)}y_N(t+1), & \text{if } t \geq 1, \\ \frac{\gamma}{2(1+\beta)}y_N(0) + \frac{1}{2} + \frac{\beta\gamma}{2(1+\beta)}y_N(1), & \text{if } t = 0. \end{cases} \quad (32)$$

Using the induction hypothesis, (19) and (32), we obtain

$$\frac{y_{N+2}(t)}{y_{N+2}(t-1)} > \frac{y_{N+1}(t)}{y_{N+1}(t-1)}, \quad 2 \leq t \leq N+1, \quad (33)$$

while using Proposition 3.3 also, we can verify that

$$\frac{y_{N+2}(1)}{y_{N+2}(0)} > \frac{y_{N+1}(1)}{y_{N+1}(0)}, \quad (34)$$

Inequalities (33) and (34) together establish (24) for  $N+1$ .  $\square$

**Proof of Proposition 3.5:** Equations (13), (16), and Proposition 3.1 imply that  $\bar{p}_N(t)$  is bounded between 0 and 1 for all  $N$  and  $t$ . This together with equations (14) and (17) imply that  $y_N(t)$  is bounded between 0 and 1 for all  $N$  and  $t$ , as well. By Proposition 3.3,  $y_N(t)$  is strictly increasing in  $N$  for any given  $t$ . Therefore, the limit on the left-hand side of (25) is well-defined and between 0 and 1. Denote this limit by  $y(t)$ . Taking  $N \rightarrow \infty$  in (32) yields

$$y(t) = \begin{cases} \frac{\gamma}{2(1+\beta)}y(t-1) + \frac{\gamma}{2}y(t) + \frac{\beta\gamma}{2(1+\beta)}y(t+1), & \text{if } t \geq 1, \\ \frac{\gamma}{2(1+\beta)}y(0) + \frac{1}{2} + \frac{\beta\gamma}{2(1+\beta)}y(1), & \text{if } t = 0. \end{cases} \quad (35)$$

Using standard methods to solve the first equation in (35), we obtain

$$y(t) = \xi y(t-1) \quad \forall t \geq 1, \quad (36)$$

where  $\xi$  is the root which solves the quadratic equation  $\xi^2 - [(1+\beta)(2-\gamma)/(\beta\gamma)]\xi + 1/\beta = 0$ , and of which the absolute value is less than 1. The root is given by (26). The second equation in (35) is simply

$$y(0) = \frac{\gamma}{2(1+\beta)}y(0) + \frac{1}{2} + \frac{\beta\gamma}{2(1+\beta)}y(1). \quad (37)$$

Substituting the relation  $y(1) = \xi y(0)$  derived from (36) into (37) yields

$$y(0) = \frac{(1+\beta)}{2(1+\beta) - (1+\beta\xi)\gamma}. \quad (38)$$

Iterating (36) on  $t$  gives  $y(t) = \xi^t y(0)$  for all  $t \geq 0$ . This together with (38) establish (25).  $\square$

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Table 1.  
The responses of the price indices

$\bar{p}_n(t)$	$n = 1$	$n = 2$	$n = 4$	$n = 8$	$n = 12$
$t = 0$	0.50	0.39	0.31	0.26	0.25
$t = 1$	1.00	0.89	0.76	0.66	0.63
$t = 2$	1.00	1.00	0.95	0.86	0.83
$t = 3$	1.00	1.00	0.99	0.95	0.93

Table 2.  
The response of real GDP

$y_N(t)$	$N = 1$	$N = 2$	$N = 4$	$N = 8$	$N = 12$
$t = 0$	0.50	0.61	0.69	0.74	0.75
$t = 1$	0	0.11	0.24	0.34	0.37
$t = 2$	0	0	0.06	0.14	0.17
$t = 3$	0	0	0.01	0.05	0.07
$y_N(1)/y_N(0)$	0	0.19	0.35	0.46	0.50

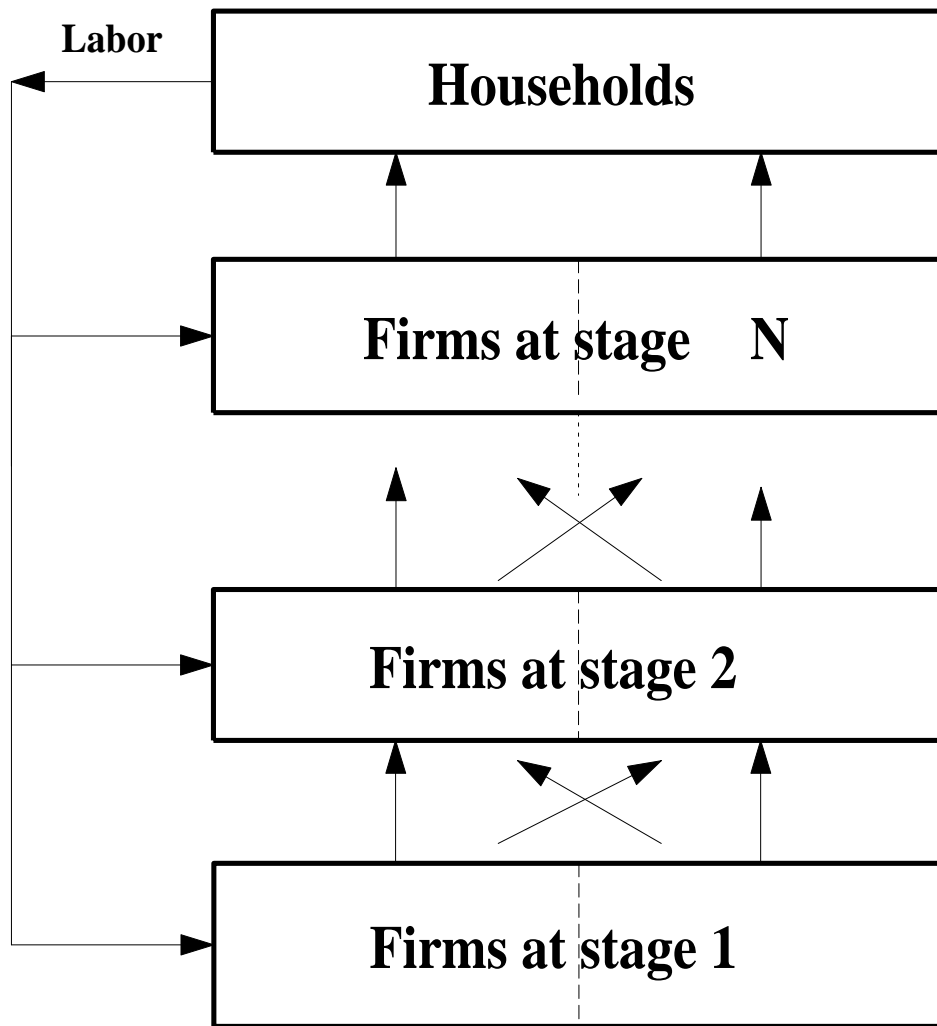


Figure 1:—The input-output structure of the economy

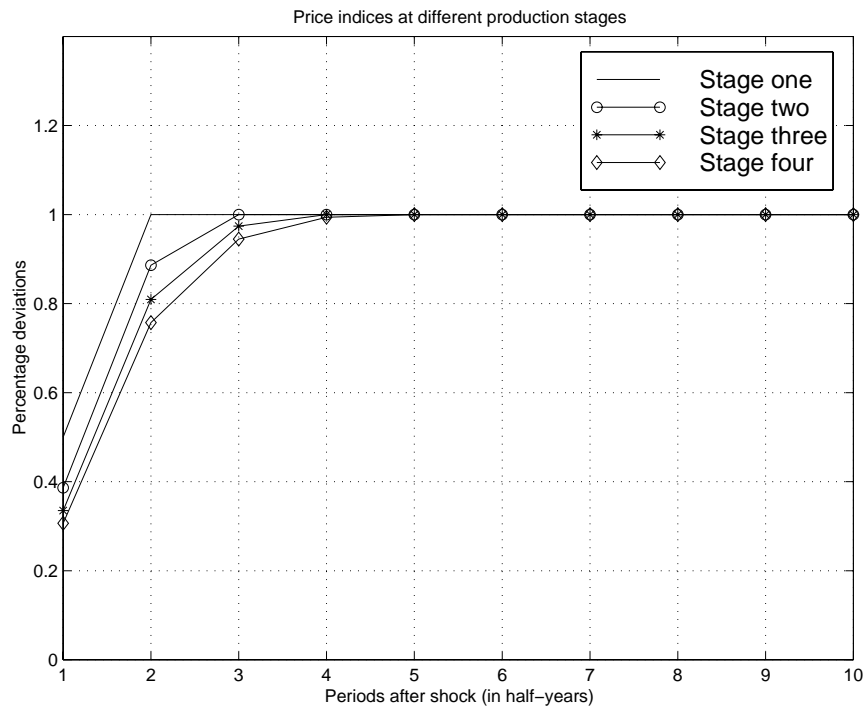
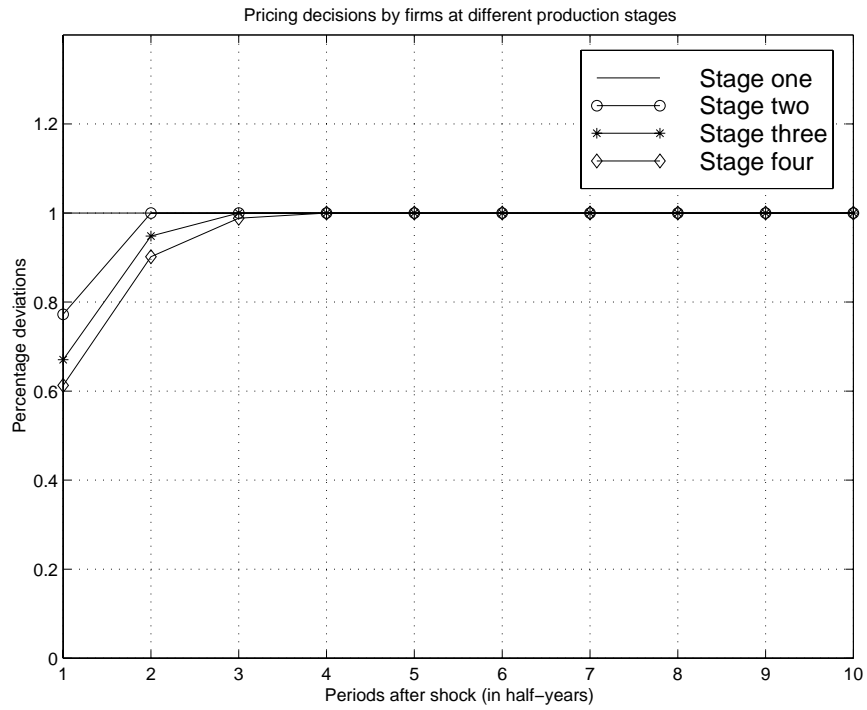


Figure 2:—The responses of pricing decisions and the price indices at different intermediate stages of production (with 2 cohorts of price-setting firms at each stage)

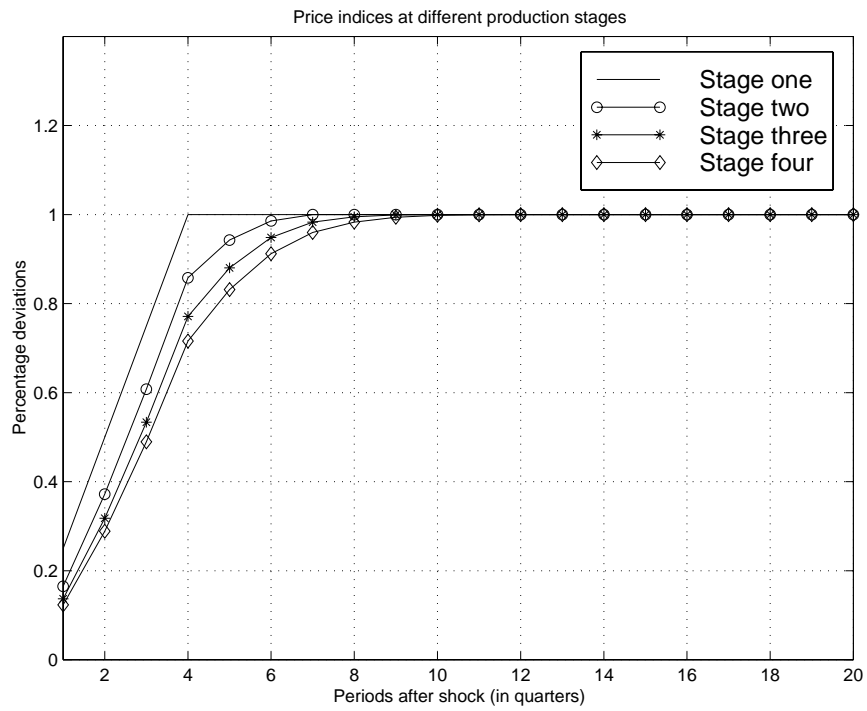
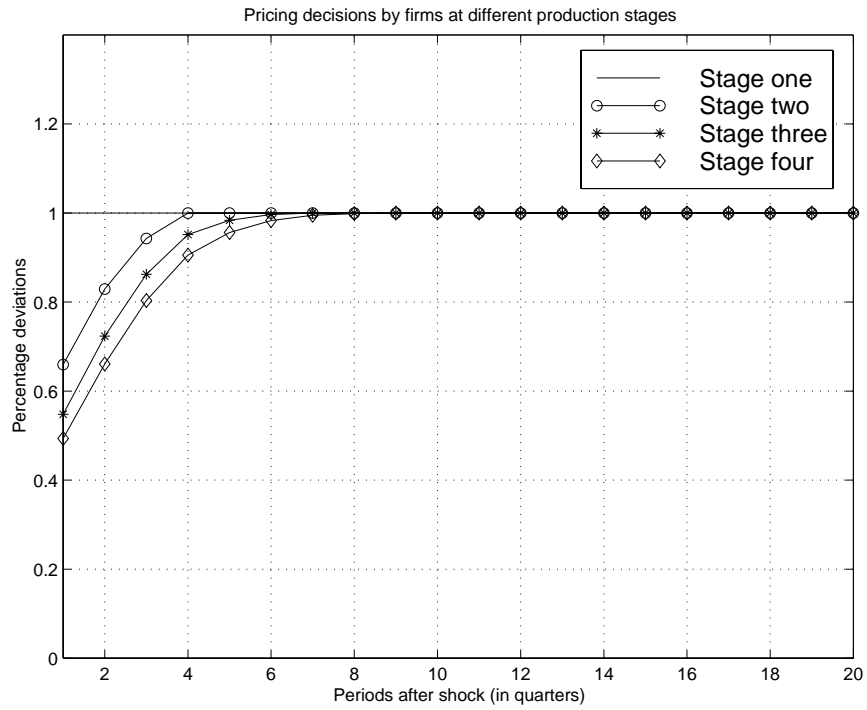


Figure 3:—The responses of pricing decisions and the price indices at different intermediate stages of production (with 4 cohorts of price-setting firms at each stage)

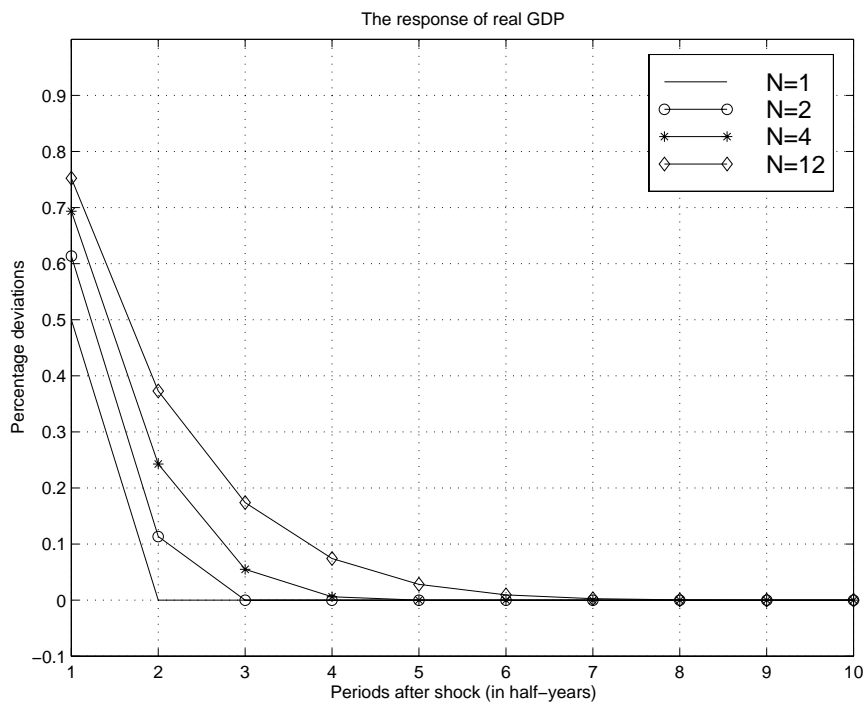
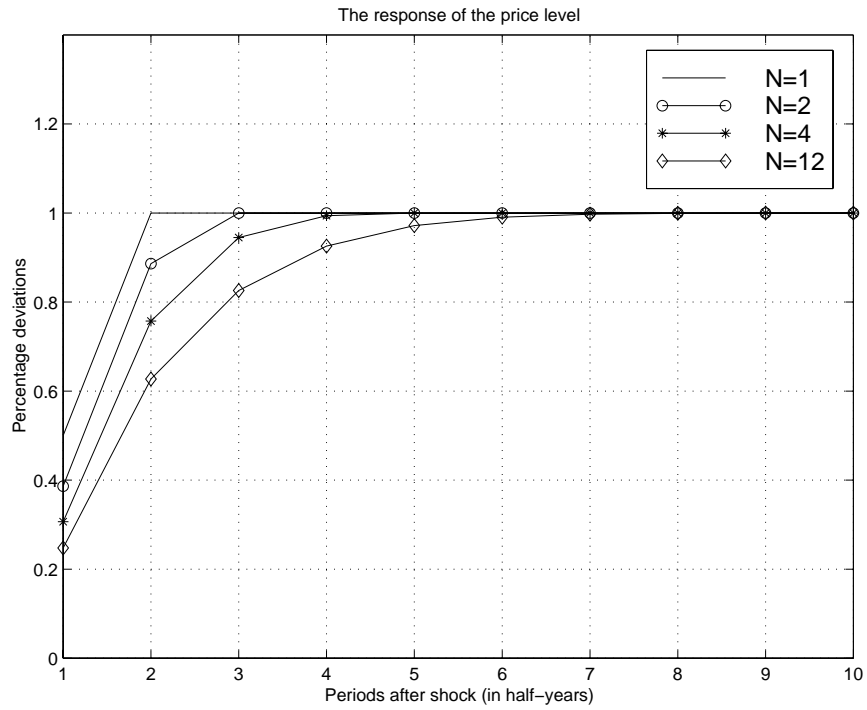


Figure 4:—The responses of the price level and real GDP for different number of stages of production (with 2 cohorts of price-setting firms at each stage)

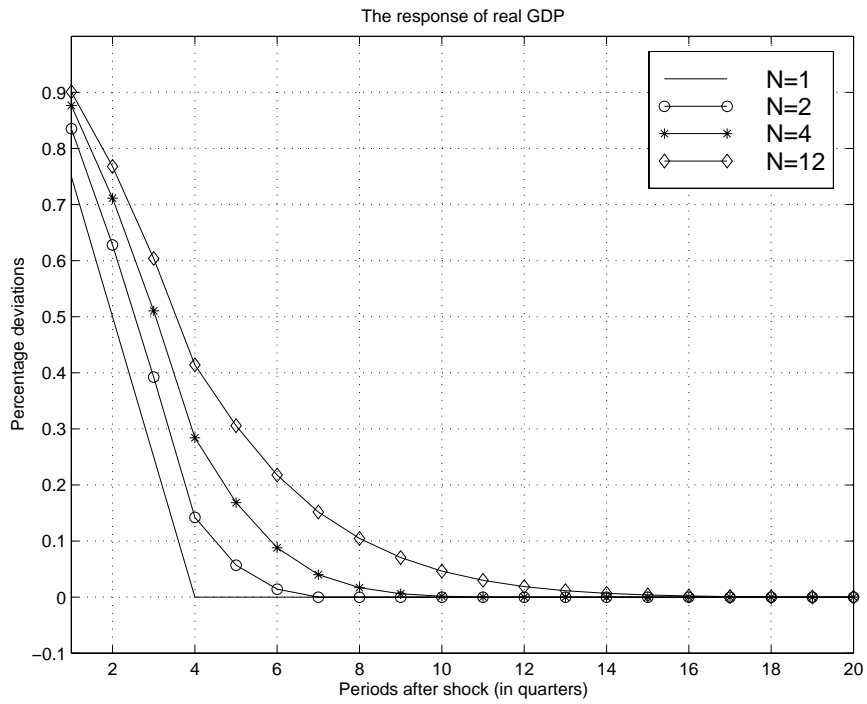
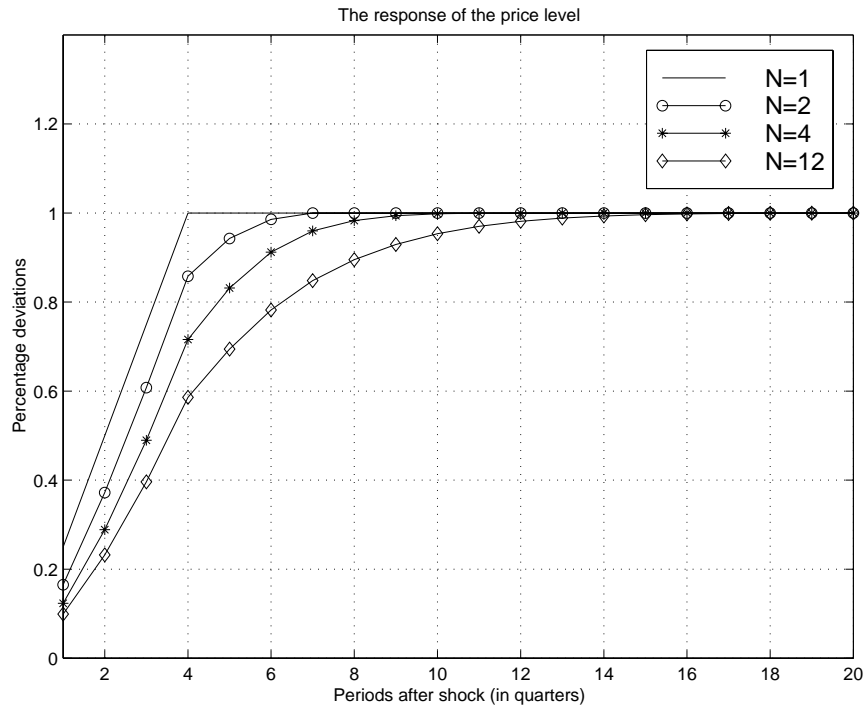


Figure 5:—The responses of the price level and real GDP for different number of stages of production (with 4 cohorts of price-setting firms at each stage)

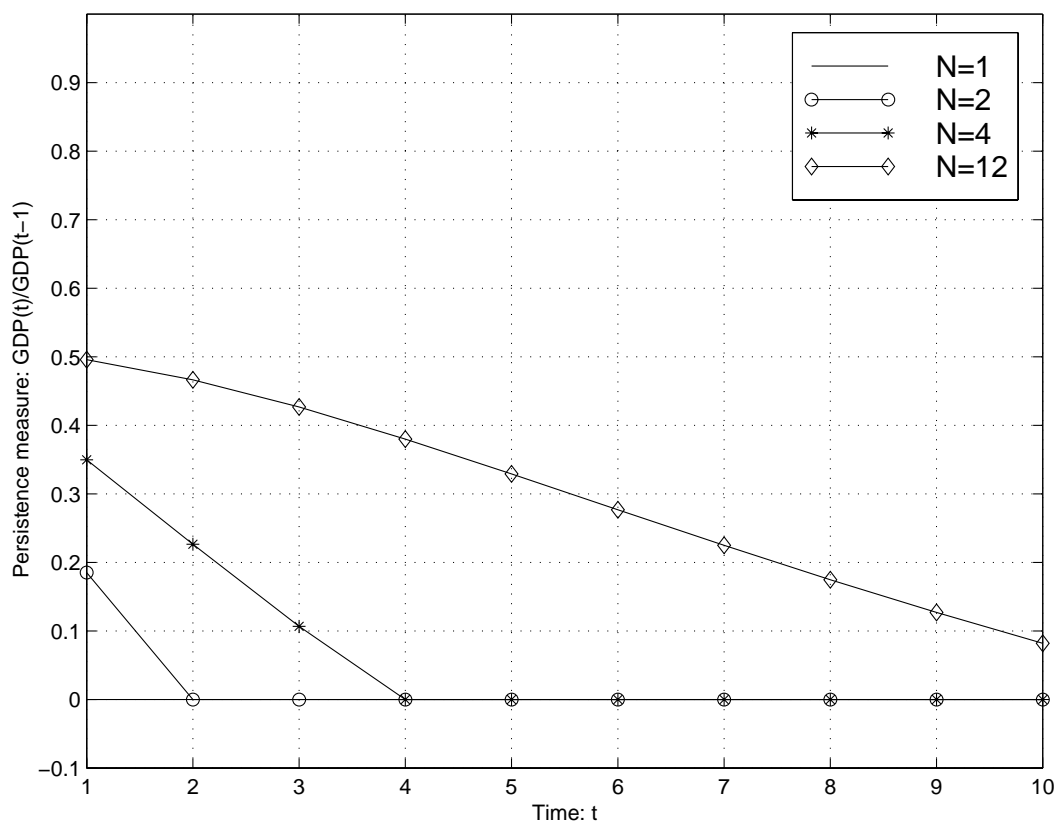


Figure 6:—The monotonic relation between the persistence measure (i.e.,  $y_N(t)/y_N(t-1)$  for  $1 \leq t \leq N$ ) and the number of stages of production ( $N$ ) (with 2 cohorts of price-setting firms at each stage)

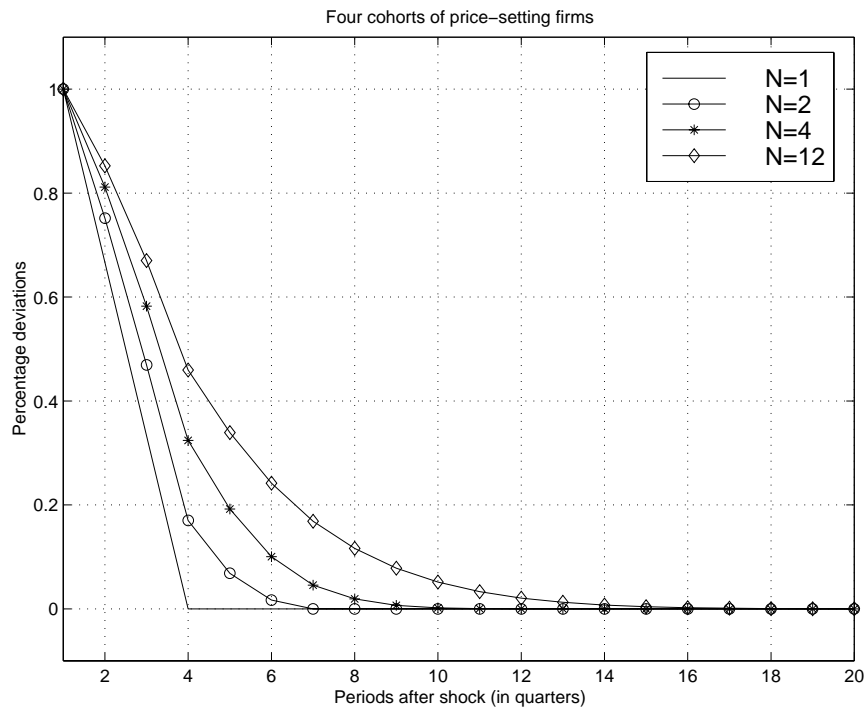
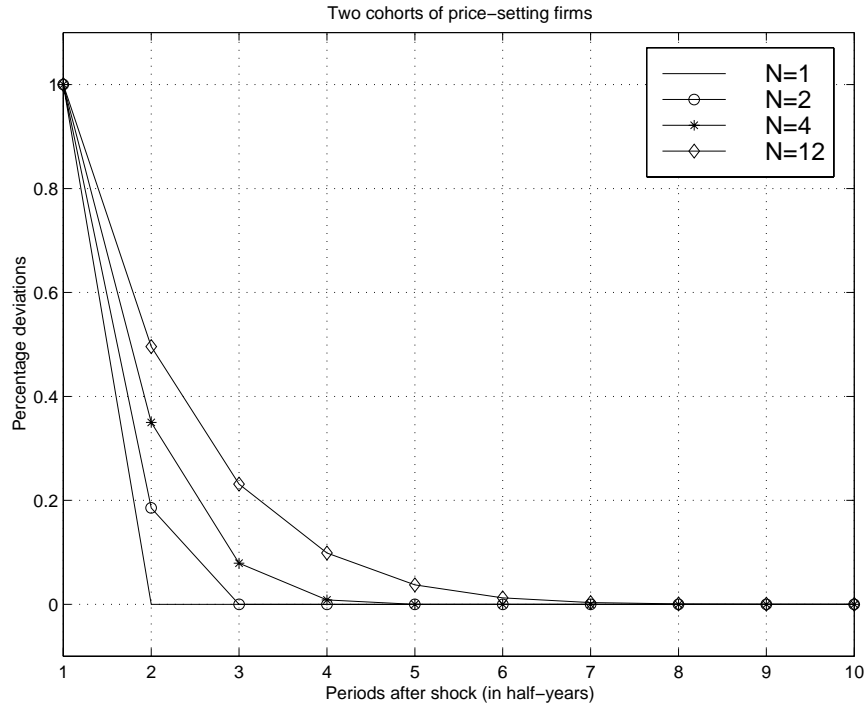


Figure 7:—The response of real GDP relative to its initial response